Key

Math 4 Name ______________________

**4-7 Sum and Difference Formulas for Sine and Tangent** Date________

*I can, without a calculator, use trigonometric identities such as angle addition/subtraction and double angle formulas, to express values of trigonometric functions in terms of rational numbers and radicals.*

**Recall:**

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

**But wait, there's more.... The cosine sum formulas can be used to derive ones for sine.**

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

**And the sine and cosine sum formulas can be used to derive ones for tangent.**

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

$$\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

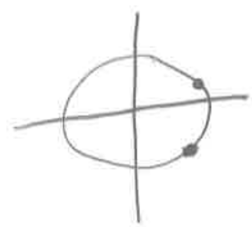
**Now you try. Find the exact values for the following.** → Use the equations above + your knowledge of the Unit Circle!

**1.** $\sin(310^\circ)\cos(5^\circ) + \cos(310^\circ)\sin(5^\circ) =$

$= \sin(310^\circ + 5^\circ)$

$= \sin 315^\circ$

$= \boxed{-\frac{\sqrt{2}}{2}}$



**2.** $\tan\left(\frac{7\pi}{12}\right)$

$= \tan\left(\frac{3\pi}{12} + \frac{4\pi}{12}\right)$

$= \tan\left(\frac{\pi}{4} + \frac{\pi}{3}\right)$

$= \dfrac{\tan\left(\frac{\pi}{4}\right) + \tan\left(\frac{\pi}{3}\right)}{1 - \tan\left(\frac{\pi}{4}\right)\cdot\tan\left(\frac{\pi}{3}\right)}$

$= \dfrac{1+\sqrt{3}}{1 - 1\cdot\sqrt{3}}$

$= \boxed{\dfrac{1+\sqrt{3}}{1-\sqrt{3}}}$

**3.** $$\frac{\tan\left(\frac{3\pi}{16}\right)+\tan\left(\frac{\pi}{16}\right)}{1-\tan\left(\frac{3\pi}{16}\right)\tan\left(\frac{\pi}{16}\right)}$$

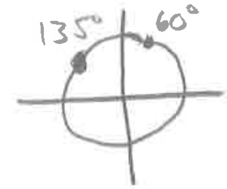
$$=\tan\left(\frac{3\pi}{16}+\frac{\pi}{16}\right)$$

$$=\tan\left(\frac{4\pi}{16}\right)$$

$$=\tan\left(\frac{\pi}{4}\right)$$

$$=\boxed{1}$$

**4.** $\sin(195^\circ)$




$$=\sin(60^\circ+135^\circ)$$

$$=\sin(60)\cdot\cos(135)+\cos(60)\sin(135)$$

$$=\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}+\frac{-1}{2}\cdot\frac{\sqrt{2}}{2}$$

$$=\frac{\sqrt{3}\cdot\sqrt{2}}{4}+\frac{-\sqrt{2}}{4}$$

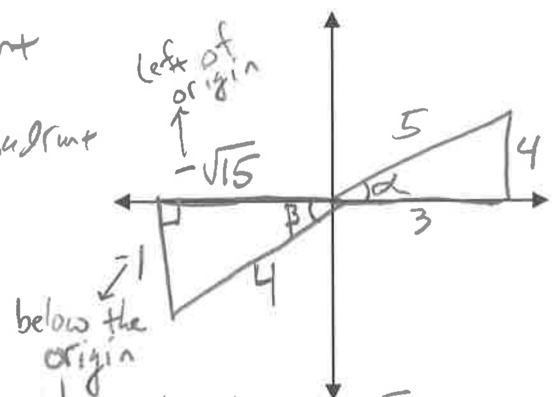
$$\sqrt{3}\cdot\sqrt{2}=\sqrt{3\cdot 2}=\sqrt{6}$$

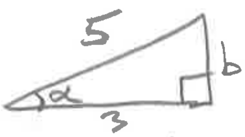
$$=\frac{\sqrt{6}}{4}-\frac{\sqrt{2}}{4}=\boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$$

**5.** **Given:** $\cos\alpha=\frac{3}{5},\ 0<\alpha<\frac{\pi}{2}$ 1st Quadrant

$\sin\beta=-\frac{1}{4},\ \pi<\beta<\frac{3\pi}{2}$ 3rd Quadrant

**Find:** $\sin(\alpha-\beta)$, $\cos(\alpha-\beta)$, $\tan(\alpha-\beta)$




$\cos\alpha=\frac{3}{5}=\frac{\text{Adjacent}}{\text{Hypotenuse}}$

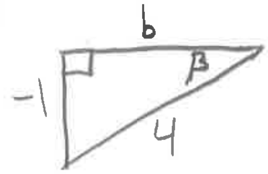
$3^2+b^2=5^2$

$b=4$

So,

$\sin\alpha=\frac{4}{5}=\frac{\text{Opposite}}{\text{Hypotenuse}}$

$\tan\alpha=\frac{4}{3}=\frac{\text{Opposite}}{\text{Adjacent}}$




$(-1)^2+b^2=4^2$

$b^2=15$

$b=-\sqrt{15}$

$\cos\beta=\frac{-\sqrt{15}}{4}$

$\sin\beta=\frac{-1}{4}$

$\tan\beta=\frac{-1}{-\sqrt{15}}=\frac{1}{\sqrt{15}}$

$$\sin(\alpha-\beta)=\frac{4}{5}\cdot\frac{-\sqrt{15}}{4}-\frac{3}{5}\cdot\frac{-1}{4}$$

$$=\frac{-4\sqrt{15}}{20}+\frac{3}{20}$$

$$=\boxed{\frac{-4\sqrt{15}+3}{20}}$$

$$\cos(\alpha-\beta)=\frac{3}{5}\cdot\frac{-\sqrt{15}}{4}+\frac{4}{5}\cdot\frac{-1}{4}$$

$$=\frac{-3\sqrt{15}}{20}-\frac{4}{20}$$

$$=\boxed{\frac{-3\sqrt{15}-4}{20}}$$